Imperial College London

DE2 Electronics 2

Lecture 17

Revision Lecture

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10 things you have learned about signals (1)

1. Signals can be represented in **time domain** or **frequency domain**.
2. Any signal can be made up from **weighted sum of sinusoidal** signals.
3. A sinusoid at frequency ω and amplitude A can be an everlasting sine wave ($A \sin \omega t$), cosine wave ($A \cos \omega t$) or exponential ($A/2\, e^{j\omega t}$). Furthermore, two sinusoids at different frequencies have **NOTHING in common**.
4. For a **time-limited** signal, moving between time and frequency domain is done through **Fourier Transform**.
5. A **periodic signal** is represented in the frequency domain in **Fourier series**, where the fundamental frequency f_0 is 1/period of the signal, and all the other frequency are integer multiple of f_0.

10 things you have learned about signals (2)

6. You must sample a signal at a sampling frequency f_s which is **at least twice** that of the maximum signal frequency f_{max}: $f_s \geq 2*f_{max}$.
7. When **sampling signal** at f_s, the **spectrum** of the original signal is **repeated** at EVERY multiple of sampling frequency, i.e $\pm nf_s$, n = 1, 2, 3...
8. If you sample a signal which has a frequency component higher than fs/2, **aliasing** occurs (which results in **spectral folding**).
9. When you **extract** a portion of a signal, you effectively multiply the signal with a **rectangular window**, which results in spreading of energy to neigbouring frequency components. This is known as "**leakage**".
10. You can **reduce** this **leakage** by multiplying your signal with a **special window** function which has smooth instead of shape edges.

Q1 Basic Signals

1. An electrical signal is represented mathematically by the equation:

$$x(t) = 2.35\sin(3142t + 30°) + 0.65 \text{ volts}$$

~1min/mark

(i) What is the average value of $x(t)$?

0.65v

[2]

(ii) What is the frequency in Hz and phase angle in radians of the signal?

500Hz, $\frac{\pi}{6}\ rad/\sec\ at\ t = 0$

[3]

(iii) What are its maximum and minimum values?

[4]

3v and -1.7v

Q1 – Basic signals

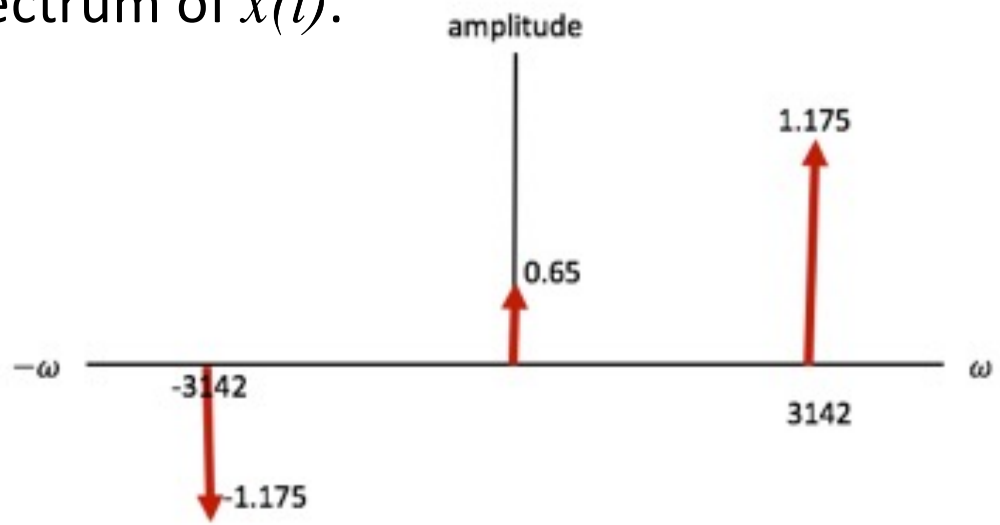
$$x(t) = 2.35\sin(3142t + 30°) + 0.65 \text{ volts}$$

(iv) Rewrite x(t) in exponential instead of sinusoidal form.

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$x(t) = 0.65 + \frac{1.175}{j}\left(e^{j(3142t+\frac{\pi}{6})} - e^{-j(3142t+\frac{\pi}{6})}\right)$$

(v) Sketch the amplitude spectrum of *x(t)*.



Q2 – Sampling & ADC

The signal in Q1 is to be sampled by a microprocessor system with an analogue to digital converter (ADC).

(i) What is the minimum sampling frequency that you must use in order not to corrupt the converted signal? In practice, what sampling frequency would you choose to use and why?

Fs(min) = 1kHz,

2.5 x Fs(min) or higher, i.e. 2.5kHz.

(ii) It is known that the converted digital signal requires an accuracy of 0.1%. How many bits of resolution is required of the ADC to achieve this accuracy?

requires 10 bits ADC – 1 part in 1024.

Q3 (i) – Sampling & ADC

You are designing an electronic system to measure breathing of a patient with a breathing frequency of up to 40 breaths/minute. However, the diagnostic technique measures turbulence in the flow that would be up to 100 times faster than the breathing frequency. Furthermore, the medical team told you that they need your system to have a measurement accuracy of 0.05%. You are required to pick an analogue to digital converter (ADC) to turn your transducer measurements, which ranges from 0 to 3.3V, to digital samples.

(i) What would you choose as the sampling frequency for the ADC? Why?

(i) Breathing rate is 40 breath/min or 0.667, and we want to capture signals at least 100 times faster. Maximum frequency is about 66.7. Sampling theorem dictates that the sampling rate must be at least 2x of maximum frequency. So, must use sampling rate of 133Hz at least. However, for easy reconstruction, we usually use a practical sampling rate somewhat higher than this (say 2.5x) at 500Hz. Accept well justified answers.

Q3 (ii) – Sampling & ADC

You are designing an electronic system to measure breathing of a patient with a breathing frequency of up to 40 breaths/minute. However, the diagnostic technique measures turbulence in the flow that would be up to 100 times faster than the breathing frequency. Furthermore, the medical team told you that they need your system to have a measurement accuracy of 0.05%. You are required to pick an analogue to digital converter (ADC) to turn your transducer measurements, which ranges from 0 to 3.3V, to digital samples.

(ii) How many bits must the ADC have as converted values? Why? What is the resolution of the ADC in volts?

0.05% = 1 in 2000. Therefore, minimum number of bits in ADC is 11-bits (2^{11}). The resolution of the ADC in volts would be 3.3V/2048 = 1.61mV.

Q3 (iii) – Anti-aliaising

You are designing an electronic system to measure breathing of a patient with a breathing frequency of up to 40 breaths/minute. However, the diagnostic technique measures turbulence in the flow that would be up to 100 times faster than the breathing frequency. Furthermore, the medical team told you that they need your system to have a measurement accuracy of 0.05%. You are required to pick an analogue to digital converter (ADC) to turn your transducer measurements, which ranges from 0 to 3.3V, to digital samples.

(iii) You were also told that the transducer could pick up interference of unknown frequencies from its surrounding. State with justifications and assumptions how you may avoid your captured signal being corrupted by such interference.

(iii) To prevent corrupting our base-band signal due to aliasing effect or frequency folding due to sampling, we need to first filter the signal with an anti-aliasing filter. If we choose a sampling frequency of 500Hz, then we need to use a lowpass filter on the breathing signal before sampling that cut out all signals (at least 40dB attenuation) for signals above 250Hz.

9 Things you learned about Systems (1)

1. Laplace transform is useful for analysing systems. It maps time domain behaviour to the complex frequency s-domain where $s = \alpha + j\omega$. This contrasts with Fourier transform which maps to frequency (or ω) domain.

2. Laplace transform converts mathematical models of real systems described using differential equations in time domain to algebraic equation in s-domain. This is possible because:

$$\mathcal{L}\left(\frac{d}{dt}\right) = s \quad \text{and} \quad \mathcal{L}\left(\frac{d^2}{dt^2}\right) = s^2$$

3. Transfer function of a system H(s) is the Laplace transform of the output signal Y(s) divided by the Laplace transform of the input signal X(s):

$$H(s) = \frac{Output\ Y(s)}{Input\ X(s)}$$

9 Things you learned about Systems (2)

4. **Accelerometer** measurement of angle is inherently **noisy** - it cannot distinguish acceleration due to gravity or due to motion.

5. **Gyroscope** measurement of angle is inherently ”**drifty**” – gyroscope provides angular velocity measurement. Angle measurement is derived through integration. This results in time varying offset called drift.

6. Much better angle estimation can be obtained by **filtering and fusion** of the two types of measurements.

9 Things you learned about Systems (3)

7. PWM is the efficient way to drive motors or LEDs. The H-bridge motor driver allows PWM signal to control the speed with separate digital signals to control the direction of the motor.
8. Interrupt is a much better way of detecting hardware events than using polling method.
9. Interrupt makes software hard to debug because once set up, it runs in the background all the time and is difficult to stop. So make interrupt service routine as simple as possible.

Q4 – Motor sensor, Polling vs interrupt

Figure Q3 shows a motor with a hall effect sensor detecting the rotational speed of a motor. The circular magnet attached to the motor's axle has 12 pairs of magnetic poles. The output of the hall effect sensor produces a series of pulses, one pulse for every N-S pair of the circular magnet, which is counted by the microprocessor system.

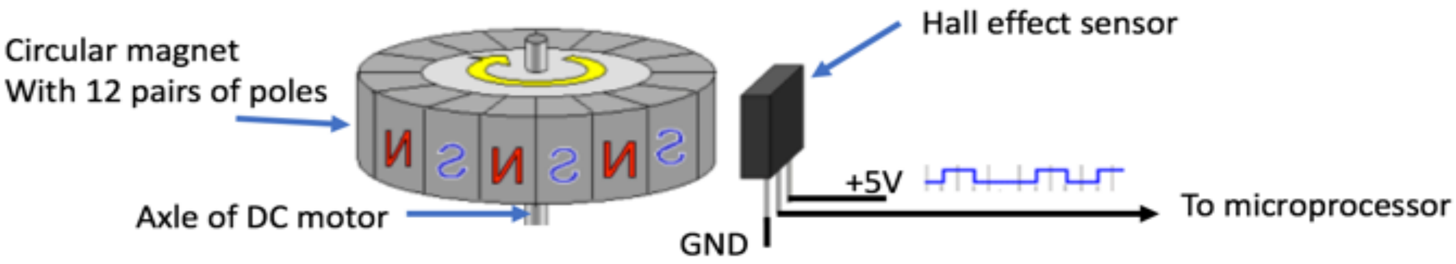
(i) If C is the number of pulses counted over a period of 100 msec, write down the equation relating the speed of the motor S in revolution/minute (rpm) to the pulse count C.

$$S = \left(\frac{C}{12} \times 10\right) \times 60 \text{ rpm}$$

[6]

(ii) The pulses could be counted by the microprocessor using the method of poling or interrupt. Explain in no more than 100 words the advantages and disadvantages of these two methods.

[6]



Q5 – Differential equation & Laplace Transform

The following differential equation describes the relationship between the output y(t) and the input x(t) of a linear system:

$$7\frac{d^2y}{dt^2} + \frac{dy}{dt}10 = \frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 4y(t)$$

(i) What is the order of this system?

This is a second order system.

[2]

(ii) Given that Y(s) and X(s) are the Laplace Transforms of y(t) and x(t) respectively, write down the transfer function H(s) = Y(s)/X(s) for the system.

[4]

$$H(s) = \frac{s^2 + 2s}{7s^2 + 10s + 4}$$

Q6 (i) & (ii) – Transfer function

A system H consists of two circuits A and B connected in series with each other as shown in Figure Q5. The transfer function for circuit A is P(s) and for circuit B is Q(s), and they are known to be:

$$P(s) = \frac{1}{0.5s + 1} \qquad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

where k is a constant.

(i) Derive the s-domain equation for the transfer function H(s) of the entire system?

$$H(s) = P(s)Q(s) = \frac{100}{0.5s^3 + (k+1)s^2 + (2k+50)s + 100}$$

(ii) What is the natural or resonant frequency of the system?

$$K \frac{\omega_0{}^2}{s^2 + 2\zeta\omega_0 s + \omega_0{}^2}$$

The natural frequency is determined by the Q(s) and is $\sqrt{100}$ = 10 rad/sec. [3]

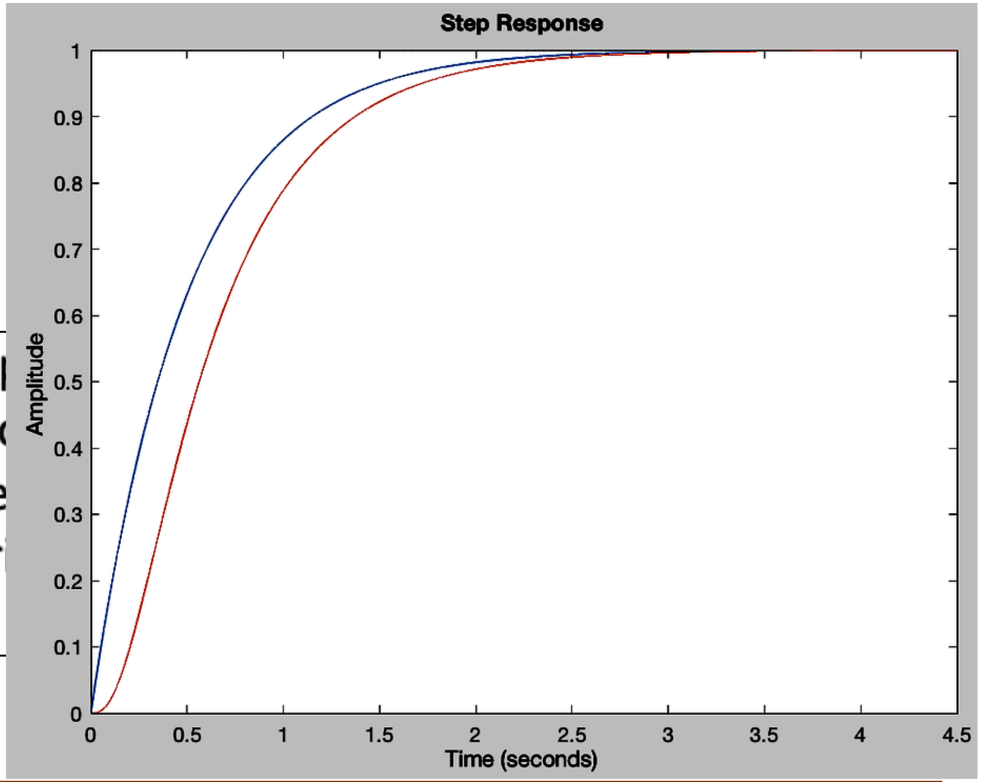
Q6 (iii) – Step Response & critical damping

$$P(s) = \frac{1}{0.5s + 1} \qquad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

$$K \frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2}$$

(iii) It is known that when k = 10, the system is critically damped. Sketch the step response of the system. Explain your answer.

When the system is critically damped, the step
dominated by the first-order system P(s), whic
constant of 0.5sec (shown in blue). Therefore
response is approximated by an exponential r
roughly 63% at 0.5s. (Shown in blue).



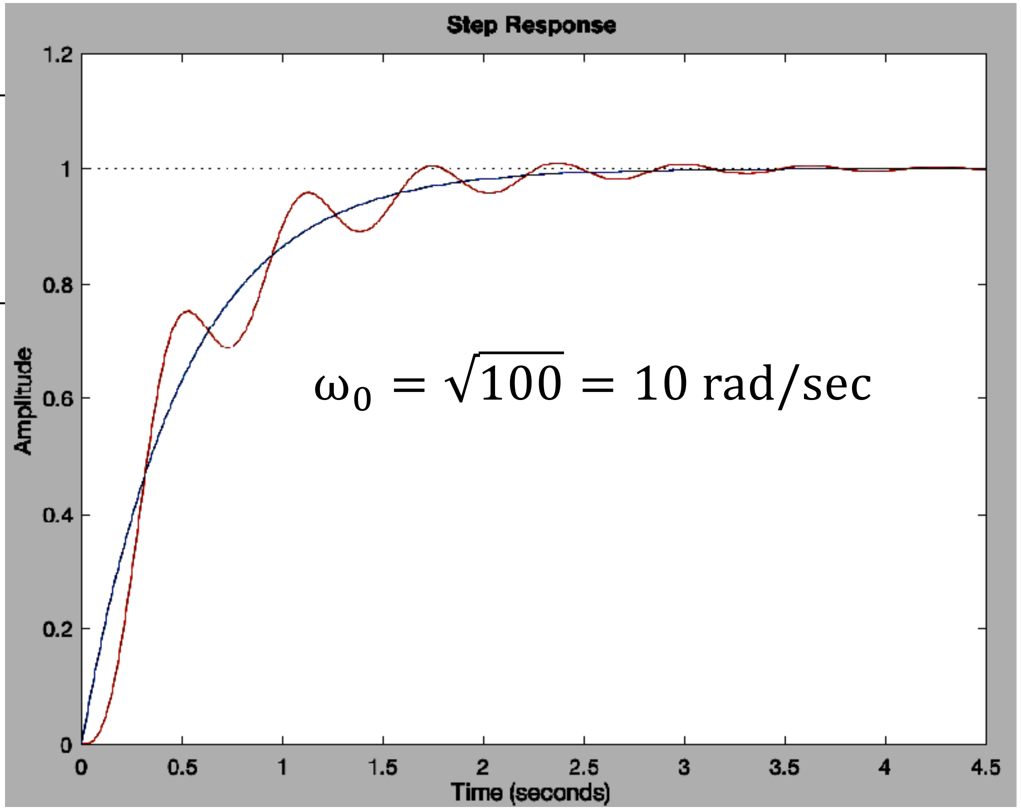
Q6 (iv) – Step Response & underdamping

$$P(s) = \frac{1}{0.5s + 1} \qquad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

$$K \frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2}$$

(iv) If k is 1, sketch the step response of the system. Explain your answer.

When k = 1, the system is underdamped, and therefore you will see oscillation at the natural frequency, which is around 1.6Hz.



$$\omega_0 = \sqrt{100} = 10 \text{ rad/sec}$$

6 Things about Discrete Time Signals & Systems (1)

1. A discrete-time system can be represented in three ways:
   - **Block diagram**
   - **Difference equation**, e.g. $y[n]$= 0.25 (x[n] + x[n-1]+x[n-2]+x[n-3])
   - **Impulse response** in the **z-domain**, e.g. $H[z] = \frac{1}{4}(1 + z^{-1} + z^{-2} + z^{-3})$

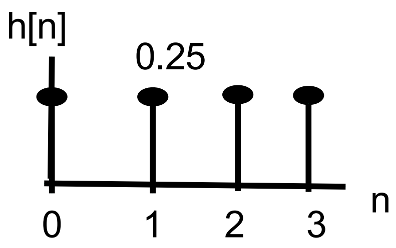
2. A **Finite Impulse Respo**nse filter (FIR), as the name implies, is one where the effect of input signal lasts for fixed number of sampling periods before reaching zero. However **Infinite Impulse Respnse** (IIR) filter in theory has an infinite impulse responses the lasts forever, but may decay towards zero.

3. An **Infinite Impulse Respnse** (IIR) filter is much more efficient to implement than FIR filter. For the same filtering effect, IIR filter has much lower number of multiplier and add operations than FIR filter. However, if badly designed IIR filter may be unstable.

6 Things about Discrete Time Signals & Systems (2)

4. A discrete time system can be characterized by its impulse response:

$$h[n] = b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2] \ldots. + b_k\delta[n-k]$$



Impulse response of a 4-tap moving average filter

5. Once we know the impulse response $h[n]$, and the input sequence $x[n]$, we can find the output $y[n]$ by convolution:

$$y[n] = x[n] * h[n] = \sum_{m=0}^{\infty} x[m]h[n-m]$$

6 Things about Discrete Time Signals & Systems (3)

6. Convolution operation can be performed in four steps:

   To obtain output y[n]:

   a) Reflect impulse response at n to get h[n-m]
   b) Multiply input sequence x[m] with h[n-m]
   c) Sum the product of the two sequences to get one output y[n]

$$y[n] = \sum_{m=0}^{\infty} x[m]h[n-m]$$

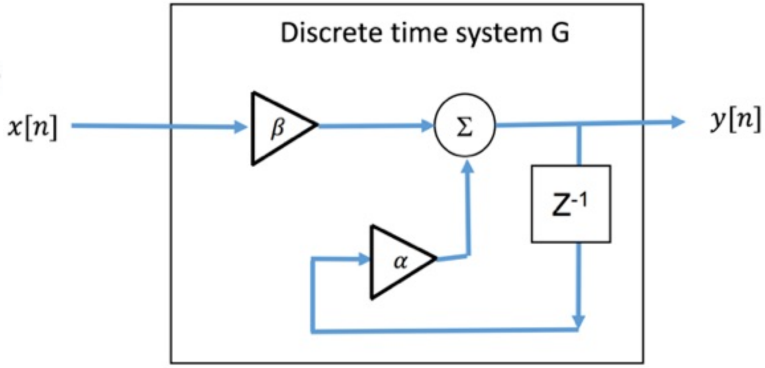
   d) Advance the reflected impulse response by one sample period and repeat to get the next y[n]

Q7 i) & (ii) – Discrete signals and z-transform

The block diagram of a discrete-time shift-invariant system G is shown in Figure Q6. The input to the system is $x[n]$ and the output is $y[n]$, where $n = 0,1,2,3,\dots$ etc. The system is assumed to be casual, i.e. $x[n] = y[n] = 0,$ for $n < 0$.

(i) Derive the difference equation for the system.

$$y[n] = \alpha y[n-1] + \beta x[n]$$



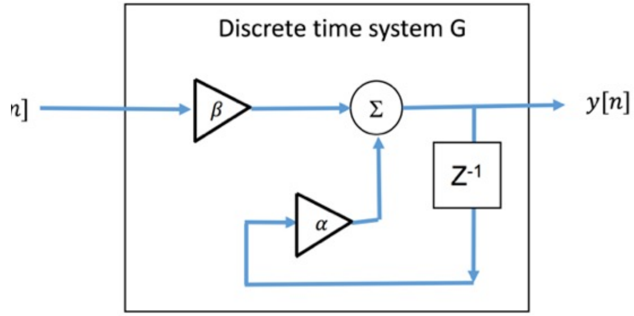
(ii) Derive the output sequence $y[0], y[1], \dots\dots\dots, y[9]$ given that $\alpha = 0.8, \beta = 0.2,$ $x[0] = 0$ and $x[n] = 10$ for $n \geq 1$.

n	0	1	2	3	4	5	6	7	8	9
x[n]	0	10	10	10	10	10	10	10	10	10
y[n]	0	2	3.6	4.88	5.9	6.72	7.38	7.9	8.32	8.66

Q7 iii) – z-transform and z-domain transfer function

The block diagram of a discrete-time shift-invariant system G is shown in Figure Q6. The input to the system is $x[n]$ and the output is $y[n]$, where $n = 0,1,2,3, \dots$ etc. The system is assumed to be casual, i.e. $x[n] = y[n] = 0,$ for $n < 0$.

(iii) Find the transfer function $G[z]$ of the system for $\alpha = 0.8, \beta = 0.2$.



$$y[n] = \alpha y[n-1] + \beta x[n]$$

Take z-transform on both sides of the difference equation:

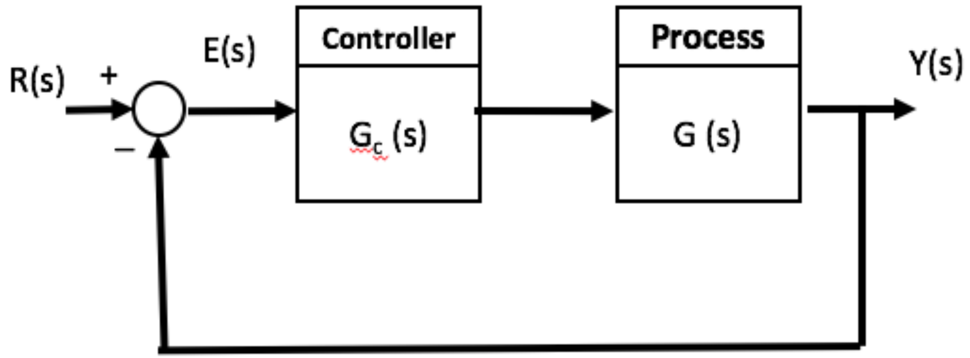
$$Y(z) = 0.8z^{-1}Y(z) + 0.2X(z)$$

$$\Longrightarrow (1 - 0.8z^{-1})Y(z) = 0.2X(z)$$

$$G(z) = \frac{Y(z)}{X(z)} = \frac{0.2}{1 - 0.8z^{-1}} = \frac{0.2z}{z - 0.8}$$

7 Things about Feedback Control (1)

1. Closed-loop negative feedback system has the general form (with example):



2. Adding the controller $G_C(s)$ and closing the loop changes the system transfer function from G(s) to:

$$\frac{Y(s)}{R(s)} = \frac{L(s)}{1 + L(s)}, \qquad \text{where } L(s) = G_c(s)G(s)$$

3. A closed-loop system reduces steady-state errors and impact of perturbation by a factor of (1 + L(s)), where L(s) is the loop gain.

7 Things about Feedback Control (2)

4. If the controller is a constant gain $G_c(s) = K_p$, it is called **proportional control**. This may not be sufficient in providing good dynamic response to the system. If K_p is large, the system may not be stable.

5. One may add an integral term to the controller: $G_c(s) = K_p + \frac{K_I}{s}$. This eliminates steady state error.

6. One may also add a differential term to the controller: $G_c(s) = K_p + \frac{K_I}{s} + sK_D$. This is called a PID controller.

7. PID controller requires tuning, which determines the optimal values for $K_p, K_I,$ and K_D.

Q8 – Feedback Control (1)

A DC motor system is controlled using pulse-width modulation with a duty cycle of $x(t)$, which has a range of 0 to 1. The speed of the motor $y(t)$ is measured in revolutions per second (rps), and the transfer function $G(s) = \frac{Y(s)}{X(s)}$ of the motor is given by:

$$G(s) = \frac{K_L}{0.05s + 1}$$

where K_L is a constant with a value of 4 if the system is ideal. However, manufacturing processes cause this value to vary by ±20%.

(i) If the motor is controlled directly as an open-loop system with $x(t)$ set to 0.5 (i.e. 50% duty cycle), calculate the maximum and minimum steady-state speed of the motor.

a) Nominal speed is 2, but due to variation of K_L, this may vary by ±0.4. Therefore, the maximum and minimum speed of motor would be 1.6 to 2.4 revolutions per second.

[4]

Q8 – Feedback Control (2)

A DC motor system is controlled using pulse-width modulation with a duty cycle of $x(t)$, which has a range of 0 to 1. The speed of the motor $y(t)$ is measured in revolutions per second (rps), and the transfer function $G(s) = \frac{Y(s)}{X(s)}$ of the motor is given by:

$$G(s) = \frac{K_L}{0.05s + 1}$$

where K_L is a constant with a value of 4 if the system is ideal. However, manufacturing processes cause this value to vary by ±20%.

(ii) Sketch the response of the open-loop system if x(t) is a step function $u(t)$. What is the time constant of the system?

[5]

b) This requires students to know that: 1) the transfer function indicates a first order system; 2) its step response is an exponential rise with a time-constant equals to the coefficient value of the s term. Therefore, the time-constant is 50ms. Sketching is now very easy – the exponential signal reaches 0.63% of 4, or 2.52 at 0.05 second, and eventually reaches 4 in steady state.